PROPAGATION OF AN IONIZING MHD SHOCK WAVE IN A NONUNIFORM MEDIUM

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The motion of magnetohydrodynamic (MHD) ionizing shock waves in a nonuniform medium is of interest in connection with their possible uses, for example, in experiments on shock acceleration in a gas with decreasing density (gradient acceleration) or in a plasma with a nonuniform magnetic field, and also in connection with astrophysical applications to problems on the dynamics of a cosmic plasma.

The law governing the increase in the velocity of a shock front, as derived from the self-modeling solution for a gas with decreasing density in the absence of a magnetic field, can be written in the form $u=\rho^{-\lambda}$, where $\lambda=0.2$ [1]. In the presence of a magnetic field the acceleration of the shock front in an ideal plasma is considerably more effective within the framework of the self-modeling solution and the index is higher ($\lambda=0.5$ in [2], where the whole medium is considered to be infinitely conducting). It is of interest to consider the case when the gas is ionized by the shock wave itself.

A numerical solution has been obtained to the equations which describe the one-dimensional flow of a finite-conductivity plasma behind the front of an MHD ionizing shock wave in the single-fluid magnetohydrodynamic approximation [3]:

$$\begin{gathered}\partial\rho/\partial t+\partial(\rho v)/\partial x=0;\\ \partial H/\partial t+\partial(Hv)/\partial x=\nu\partial^2H/\partial x^2;\\ \partial v/\partial t+v\partial v/\partial x+(1/\rho)[\partial p/\partial x+(1/8\pi)\partial H^2/\partial x]=0;\\ \partial(\rho\varepsilon)/\partial t+p\partial v/\partial x+\partial(\rho\varepsilon v)/\partial x=(\nu/4\pi)(\partial H/\partial x)^2,\end{gathered}\tag{1}$$

where ρ and ε are the density and specific internal energy of the plasma; p is the gas-kinetic pressure; v is the mass velocity; H is the z component of the magnetic field ($H_x=H_y=0$); ν is the magnetic viscosity, $\nu=c^2/4\pi\sigma$; σ is the plasma conductivity; and c is the velocity of light. The relations on the shock front have the usual hydrodynamic form [1, 3]

$$\begin{gathered}\rho_1u=\rho_2(u-v_2);\\ p_1+\rho_1u^2=p_2+\rho_2(u-v_2)^2;\\ \omega_1+(1/2)u^2=\omega_2+(1/2)(u-v_2)^2,\end{gathered}\tag{2}$$

and these must be supplemented by the condition for the continuity of the magnetic field $H_1=H_2$. The subscript 1 here refers to quantities directly ahead of the shock and the index 2, to quantities behind the front; $\omega=\varepsilon+p/\rho$; u is the velocity of the shock; and ω is the specific enthalpy.

The shock wave was created by an ideally conducting piston moving at a definite velocity through a gas at rest.

We now estimate the characteristic lengths for ionization l_i and for attenuation of the magnetic field l_m. For an ionization relaxation time behind the front of $\tau\sim10^{-6}$ sec [1] (the density $N\sim10^{19}-10^{20}$ cm^{-3}) and a relative plasma velocity of $v\sim10^6$ cm/sec, we have $l_i\sim\tau v\leq1$ cm. For a magnetic viscosity $\nu\sim10^7$ cm/sec and the same characteristic velocity, the attenuation length for the magnetic field is of the order of $l_m\sim\nu/v\geq10$ cm. Thus,

$$l_i\ll l_m.\tag{3}$$

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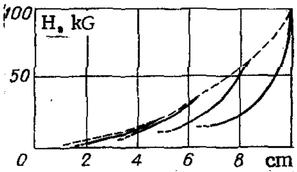

Fig. 1

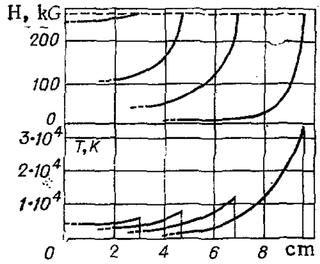

Fig. 2

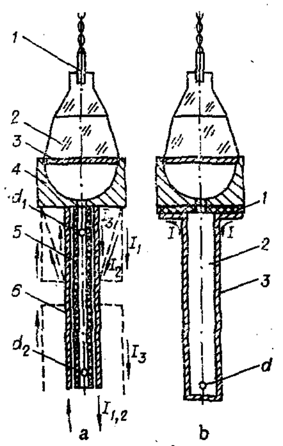

Fig. 3

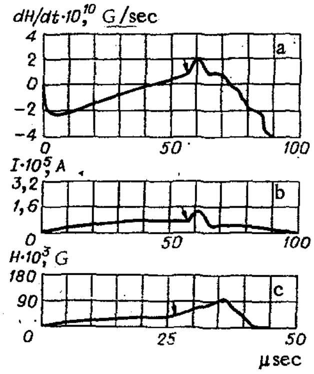

Fig. 4

We will assume in what follows that (3) is satisfied. We need not therefore consider the full kinetic equation for the ionization, but we can assume that the degree of ionization immediately takes up its equilibrium value.

The equation of state of the plasma is

$$p = (1 + \alpha)\rho RT$$

and the internal energy is

$$\varepsilon = [RT/(\gamma - 1)](1 + \alpha) + \alpha IR/k,$$

where α is the degree of ionization; T is the plasma temperature; I is the ionization potential; k is Boltzmann's constant; and R is the universal gas constant. The equilibrium Saha equation [3] was used to calculate α. The velocity of the gas in front of the shock and the degree of ionization were taken to be zero; i.e., it was assumed that $v_1 \ll u$ and $\alpha_1 \ll \alpha_2$.

The numerical solution of (1) was based on a second-order scheme with explicit treatment of the shock wave; the calculations were carried out in Eulerian coordinates. Equation (2) on the front of the ionizing MHD shock was solved by iteration. In order to check the numerical scheme and the accuracy of the method, we compared the results of a calculation for zero magnetic field (H = 0) and zero ionization ($\alpha = 0$) with the well-known hydrodynamic solution [4].

Figure 1 shows profiles of the magnetic field behind the shock front at various times for the case where the MHD shock propagates through a uniform-density gas in a transverse magnetic field with a gradient (the dashed lines indicate the original magnetic field ahead of the shock). The parameter values were taken from the experiment which is described below. The relative change in the front velocity was ~10%.

As our second example we studied the propagation of an ionizing MHD shock in a medium with decreasing density in the presence of a constant magnetic field. The initial density is

$$\rho = \begin{cases} \rho_0 & \text{for } x \leqslant 0, \\ \rho_0(1 - x/x_0)^5 & \text{for } 0 < x < x_0, \end{cases}$$

which corresponds to the free flow of a diatomic neutral gas of density ρ_0 into a vacuum [5]. This arrangement was chosen because it can be realized experimentally. Figure 2 shows the magnetic field and temperature profiles behind the shock front at various times for a gas with an ionization potential of about 13 eV and an initial pressure of $p_0 = 50$ mm Hg (hydrogen, for example). The molecular dissociation energy was not taken into account.

The experimental arrangement is shown in Fig. 3a, where d_1 and d_2 are inductive magnetic field detectors; $I_{1,2,3}$ are the directions of the currents for various configurations of the busbars. An explosive plasma compressor [6, 7] was used to create an ionizing shock wave in the gas filling the tube 5. In the initial state the tube contains air under normal conditions. The magnetic field is created by a current flowing in the copper busbars 6 placed along the tube. The direction of the field is perpendicular to the propagation of the shock front and the maximum field strength is 250 kG. The gradient of the field can be varied by using different busbar configurations.

The electrical detonator 1 which initiates the discharge of the explosive 2 with the plane front of the detonation wave is actuated in synchronism with the discharge of a capacitor bank which produces the magnetic field ($C = 1.8 \cdot 10^{-3}$ F, $U = 5$ kV). The explosion products sweep along the metallic plate 3 which compresses and heats the working gas in the hemispherical compression chamber 4. The resultant plasma enters a tube with an 8-mm internal diameter and creates an ionizing shock wave with a velocity of 20 km/sec in the air. The initial speed of the shock wave can be varied by changing the parameters of the compressor.

The thickness of the metallic walls of the compression chamber is such that the magnetic field is prevented by the skin-effect from penetrating inside the chamber, and the outflowing plasma is therefore not magnetized. The velocity of the luminous shock front is measured by means of a high-speed SFR-2M photorecorder with its slit directed along the axis of the tube. The varying magnetic fields along the tube are recorded by means of the inductive heads d_1 and d_2. The current in the busbar circuit is measured by means of a Rogovskii band and is displayed on an oscilloscope after integration.

The parameters of the gas behind the front were determined from the front velocity by means of the tables in [8] and in the operating range $u = 10$–20 km/sec were found to be as follows: $T = (1.5–4.2) \cdot 10^4$°K, $p = (1.3–4.3) \cdot 10^3$ atm, $\rho = (1.5–1.3) \cdot 10^{-2}$ g/cm^3, and $\alpha \simeq (1–60) \cdot 10^{-2}$. The conductivity behind the front was calculated according to [9] to be $\sigma = (3–9) \cdot 10^{13}$ sec^{-1}.

The characteristic size of the compression region behind the shock front in these experiments was $l \sim 1$ cm. Thus the magnetic Reynolds number can be estimated as $Re_m \simeq ul/\nu = 0.4–2.5$.

A change in the magnetic field of up to 50 kG was created over a distance of ~ 4 cm in the working gap.

It was shown photochronographically that when the ionizing shock passed through regions with a magentic field gradient the velocity of the front varied very little with the sign or magnitude of the gradient.

Figure 4 gives typical oscillograms taken from the field detector d_1 and the Rogovskii band which measured the current in the busbars and which was placed outside the explosive region. At the instant shown by the arrow, the magnetic field behind the shock front begins to decrease; this is due to the expulsion of the field by the front of the ionizing shock.

Expulsion of the magnetic field was also observed in experiments where the circuit was closed by an ionizing shock wave traveling along the channel (see Fig. 3b, where 1 is an insulating washer; 2 is a channel with a square cross section; 3 are the busbars, with their circuit closed by means of the plasma; and d is an induction detector). An increase in the magnetic field ahead of the front was recorded by d at the instant when the plasma entered the channel. The signal from the detector is shown in Fig. 4c.

The experiments which we carried out on the propagation of an MHD ionizing shock wave in a uniform gas with ρ = const and in the presence of a magnetic field gradient show agreement with the calculations made for this case to within the limits of experimental error. This is true for both the change in the velocity of the shock front and for the nature of the variation of the magnetic field strength. The change in the shock velocity caused by the magnetic field gradient is negligible.

The magnetic field is expelled from the region ahead of the MHD shock. The expulsion and dissipation of the field behind the front lead to the appearance of strong gradients in this region. The field expulsion was also noted in [10] for the case of a uniform gas and a constant field strength.

In the other situation we considered, where the density decreases in the direction of shock propagation and the magnetic field is constant ahead of the shock, the increase in the velocity of the front was considerably higher. The magnetic field was expelled from the region ahead of the front.

Since the magnetic field is almost totally expelled by an ionizing shock both in the uniform case [10] and when there are gradients in the density and the field, we have reason to suppose that this property of flux expulsion is characteristic of all ionizing shocks under any conditions.

We now compare the propagation of an ionizing MHD shock in a medium with decreasing density and H= const with the cases of an ideal plasma ($\alpha=\infty$) and of a nonconducting gas ($\alpha=0$). The temperature, density, and plasma velocity are lower behind the front of an ionizing shock wave than for an ideal plasma. The acceleration of the front is therefore less under comparable experimental conditions. Thus, for example, when the density decreases by a factor of 100 and H=200 kG, the velocity increases by 8.7 times for an ideal plasma [11] and 5.3 times for an ionizing shock (this paper). For a nonconducting gas the increase is only 1.9 times [4].

It might be noted that the mass velocity of the gas behind the front of an ionizing MHD shock is no larger than that for a nonconducting gas [12, 13]. From these results we can conclude that there is little point in using an ionizing MHD shock to obtain high temperatures and mass velocities under laboratory conditions. However, the observed expulsion of the magnetic field from the region ahead of the shock front could be used to obtain pulses of a rapidly increasing magnetic field. In our experiments, for example, we obtained an increase of 2.5 in the field over a time of 10 μsec.

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